Polynomials: Algebra bash for the win!!!

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1 Quadratics

Here is a general quadratic: $ax^2 + bx + c = 0$. Their roots can be found using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. But why does the quadratic formula work?
Let's start with a general quadratic: $ax^2+bx+c = 0$. Divide through by a to get $x^2+\frac{b}{a}x+\frac{c}{a} = 0$. Completing the square gives:

The above equation should be in the form $(blah + blah)^2 = blah$. Square root both sides, and solve for x.

This should yield the quadratic formula. Yay!

2 Cubics

Let's move on to cubics! Here is the general cubic, with the x^3 coefficient already divided into the other coefficients: $x^3 + ax^2 + bx + c = 0$, and let's find a way to find it's roots! (Beware, the formula is quite hideous)
Substitute in $x = y - \frac{a}{3}$ to get rid of the squared term.

This should give you something like this: $y^3 + dy + e = 0$, where d and e are constants in terms of a, b, and c. Write these below:

$d =$

$e =$

Substitute $y = \sqrt[3]{u} - \sqrt[3]{v}$ (and yes, you have to cube $\sqrt[3]{u} - \sqrt[3]{v}$). This is to find quadratics in u and v so that we can write y as cube roots of the roots of a quadractic, something we already know how to find using the quadratic formula.

After some manipulations, you should get

$$v - u = e$$

$$uv = \frac{d^3}{27}$$

Use these two equations to find quadratic a in u and v, solve for u and v using the quadratic formula, then put u and v back in $y = \sqrt[3]{u} - \sqrt[3]{v}$, and place y back in $x = y - \frac{x}{a}$, and plug in d and e, then finally, ta-da!!!!