An introduction to induction!

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1 Whaaaaat?!?

Induction is an extremely powerful way of verifying/proving formulae. It lies on the basis of the domino effect: push over the first one, prove that any domino will push over the next in line, and you end up with an infinite number of toppled rectangles.

2 What next?

Let's say you are given this to prove:

$$1 + 2 + 3 + 4 \ldots + n = \frac{n(n+1)}{2}$$

The first step is to push over the first domino: show that it works when $n = 1$. This is the base case.

$$1 = \frac{1(2)}{2}$$

Now assume it works for any domino: assume it works when $n = k$

$$1 + 2 + 3 + 4 \ldots + k = \frac{k(k+1)}{2}$$

Now use the assumption above to prove it works for the next domino: when $n = k + 1$

$$1 + 2 + 3 + 4 \ldots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$

Commonalize the denominator and factor out a $k+1$.

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$
$$\frac{(k+1)(k+2)}{2}$$

which is what we get if we plug in a $k+1$ into $\frac{n(n+1)}{2}$! And we are done!

3 Practice

3.1

Prove that:

$$1 + 3 + 5 + \ldots + (2n-1) = n^2$$

3.2

Prove that:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3.3

Prove that:

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$$

3.4

Prove that:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$