The Γ Function and the Gaussian Integral

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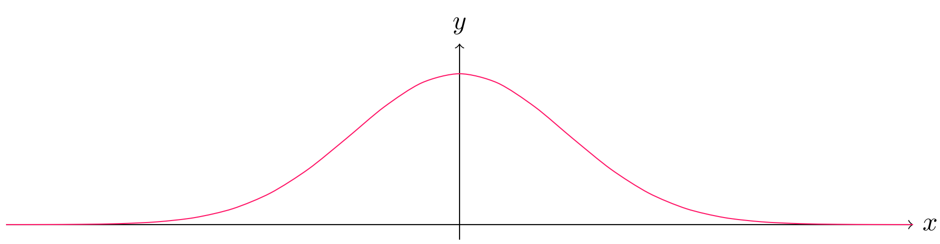
1 The Gaussian Integral

The Gaussian integral is defined to be:

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx$$

Which is an integral seen often in statistics, because $f(x) = e^{-x^2}$ is the basis of the bell curve.



Because e^{-x^2} is symmetrical across the y-axis,

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = 2\int_{0}^{\infty} e^{-x^2}\,dx$$

Changing the variable $t = x^2$, we get $\frac{1}{2\sqrt{t}}\,dt = dx$, and substituting back into the integral, we get

$$2\int_{0}^{\infty} e^{-t}\frac{1}{2\sqrt{t}}\,dt = \int_{0}^{\infty} \frac{e^{-t}}{\sqrt{t}}\,dt$$

We know from the other paper (0.5 edition) that

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty \frac{e^{-t}}{\sqrt{t}}\,dt = \sqrt{\pi}$$

And connecting the dots, we can conclude that

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$